Inductive Invariant Synthesis

Using Convex Programming and Satisfiability Modulo Theory

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VERIMAG

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Outline

Motivation

Need for Reliable Systems

- Only a couple of decades ago:
  - Computers are separate, stationary machines
- Now:
  - Hard to find a device which is not a computer
- Computerized systems can:

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  - Crash
  - Have bugs
  - Have security exploits
  - ...
- Many exploits: shellshock, heartbleed, etc.

Goal

Increasing software reliability

Static Analysis

Main Ideas

- Analyze program *without* running it

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- Sound safety proofs
  - Overflows
  - Null-pointer derefs
  - ...

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- Not complete (Turing, Church)

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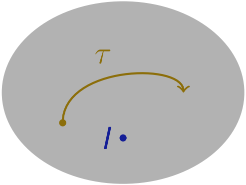
Overall Framework

Abstract interpretation unifies static analyses

Inductive Invariant

Proving Properties

- Infinite-State System
  - Proof: by induction
- Find inductive invariant
  - True by initiation
  - Holds by consecution
- Complete proof method



Everyone Loves Inductive Invariants

Verification, bug hunting, compiler optimizations, ...

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Contributions

- Theoretical
  - Policy Iteration
    - Local Policy Iteration Algorithm
      (LPI, Chapter III, VMCAI'16)
    - Template Generation Approaches
      (Chapter IV, To be Published)
    - Summary Generation Using Policy Iteration
      (Chapter V, To be Published)
  - Inductive Invariants from Preconditions
    (Chapter VI, "Formula Slicing", HVC'16)
- Engineering
  - LPI Implementation in CPAchecker
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  - Library for Utilizing Satisfiability Modulo Theory Solvers
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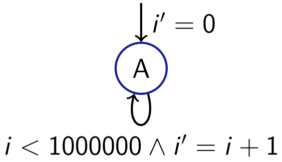
Outline

Control Flow Automaton

Program Formalization

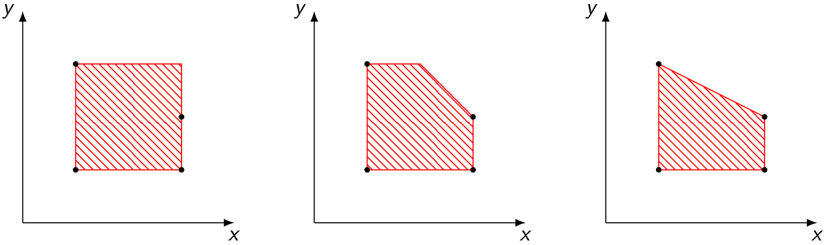
- Over program variables $\mathbf{x}$
- Transitions: associated with edges, first order formulas over
  - $\mathbf{x}$ input variables
  - $\mathbf{x}'$ output variables
- Invariants: associated with nodes, predicates over $\mathbf{x}$

```
int i=0;
while (i<1000000) {
    i++;
}
```



Abstract domains

- Domain is a lattice: set with a partial order
  - Given by inclusion
- Usual domains: intervals, octagons, polyhedra
- For a program with two variables $\mathbf{x} \equiv \{x, y\}$
  - Abstracting 4 states:



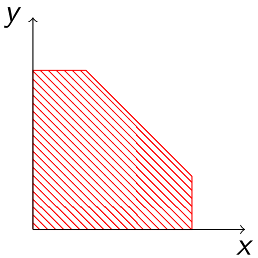
(a) intervals: $\pm x$

(b) octagons: $\pm x, \pm x \pm y$

(c) polyhedra: arbitrary convex shape $P\mathbf{x} \leq a$

Template Constraints Domain

- Polyhedra Domain:
  - most expressive
  - not a complete lattice
  - exponential runtime
- Configurable compromise: template constraints domains
  - directions fixed in advance
  - complete lattice
- For templates $T \equiv (-x, -y, x, y, x + y)$
  - State $a_0 \equiv (0, 0, 3, 3, 4)$
  - Concretizes to $0 \leq x \leq 3 \wedge 0 \leq y \leq 3 \wedge x + y \leq 4$



Template Constraints Domain

Strongest Postcondition

- Abstract semantics: transition relation in the abstract domain
- Template constraints domain: linear programming

Template Constraints Domain

Strongest Postcondition

- Abstract semantics: transition relation in the abstract domain
- Template constraints domain: linear programming

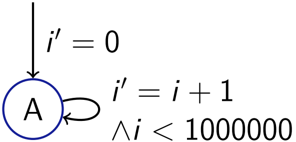
Example (Abstract Semantics)

- Template x, transition $x' = x + 1$, previous element $x \leq 5$
- New element given by $\max x'$ s. t. $x' = x + 1 \wedge x \leq 5$

Abstract Interpretation

Example Analysis in Intervals Domain

```
int i=0;
while (i < 1000000) {
    i++;
}
```



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  - $i \in [0, 0]$ (abstraction of $\{i : 0\}$)

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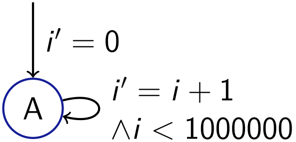

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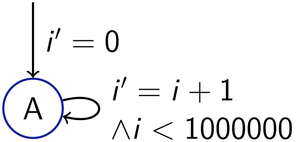

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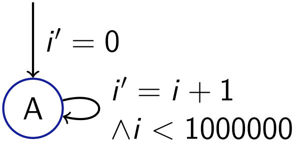

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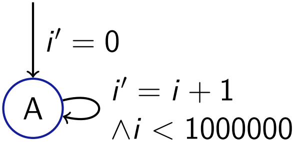

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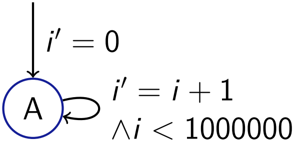

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  - $\ldots$
  - Widening: $i \in [0, +\infty)$
  - Narrowing: $i \in [0, 1000000]$

Policy Iteration

Motivation

```
int i=0;
while (input()) {
    i++;
    if (i == 1000000) {
        break;
    }
}
```

- Slightly modified program

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int i=0;
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- Narrowing is fragile

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- Slightly modified program
- Narrowing is fragile
- Narrowing: $i \in [0, \infty)$

Max-Policy Iteration

Finding *Least* Inductive Invariant

- Game-theoretic technique
- Used in artificial intelligence field
- E.g. solving poker

Max-Policy Iteration

Properties

- Generate smallest inductive invariant in the abstract domain
- Certain restriction on an abstract domain
- Exponential runtime
- Formulate as an optimization problem
- Solve non-convex optimization problem
  - By iteration over convex under-approximations (policies)

Guarantees

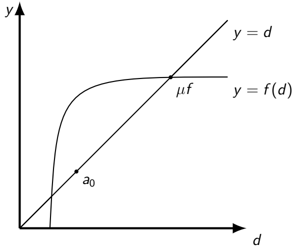
Least inductive invariant in the domain, not least invariant in general!

Max-Policy Iteration

Optimization Problem

- Simple program: one node, one initial condition, one transition
- Template Constraints Domain T, $n \equiv \|T\|$
  - Abstract state: $\mathbf{a} \in \mathbb{R}^n$
  - Abstract monotone transformer: $f : \mathbb{R}^n \to \mathbb{R}^n$
  - Initial condition: $\mathbf{a}_0 \in \mathbb{R}^n$
  - Tarski: least fixpoint of f exists in $(\mathbb{R} \cup \{+\infty, -\infty\})^n$
- Least inductive invariant definition:

$$\min \mathbf{a} \text{ s.t. } \mathbf{a} \succeq f(\mathbf{a}) \wedge \mathbf{a} \succeq \mathbf{a}_0$$



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Towards Convex Optimization Problem

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- Towards convexity: suppose f is concave
- Then *greatest* fixed point optimization problem is convex:

$$\max \mathbf{a} \text{ s.t. } \mathbf{a} \preceq f(\mathbf{a})$$

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Theorem (Fixed Point Uniqueness)

For monotone, concave f, where for initial condition a_0, $f(a_0) \succ a_0$ post-a_0 fixed point is unique!

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Introducing Policies

- What's concave?
  - Template constraints domain transfer function

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- When it stops being concave?
  - Disjunctions: multiple incoming edges
  - Each conjunct is concave
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Idea

Iterate over concave under-approximations of f (policies), find the value of each one

Max-Policy Iteration

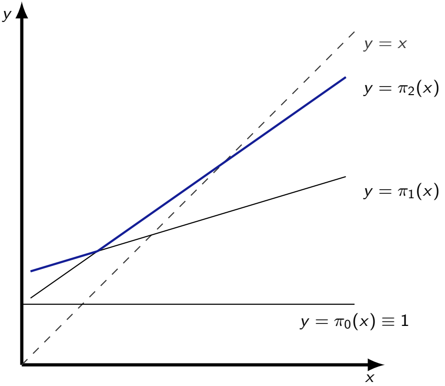
Example

```
double x = 1;
while (input()) {
  if (input()) {
     x=0.3*x+1.5;
  } else {
     x=0.7*x+1;
  }
}
```

- Find: inductive upper bound a on x
- Initial condition: $\pi_0 = \lambda d.1$
- Two policies:
  - $\pi_1 \equiv \lambda d.\max x'$ s.t. $x' = 0.3x + 1.5 \wedge x \leq d$
  - $\pi_2 \equiv \lambda d.\max x'$ s.t. $x' = 0.7x + 1 \wedge x \leq d$
- $f \equiv \lambda d.\max\{\pi_0(d), \pi_1(d), \pi_2(d)\}$
- Inductive upper bound is:
  - $\min d$ s.t. $d \geq f(d)$

Max-Policy Iteration Visualization

$$d \geq \max \begin{cases} \max x' \text{ s.t. } x' = 1 \\ \max x' \text{ s.t. } x' = 0.3x + 1.5 \wedge x \leq d \\ \max x' \text{ s.t. } x' = 0.7x + 1 \wedge x \leq d \end{cases}$$



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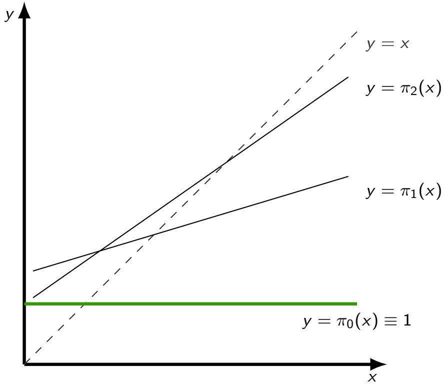

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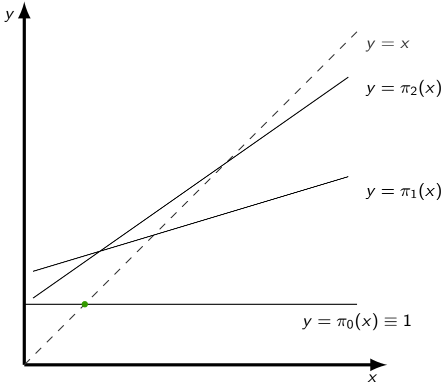

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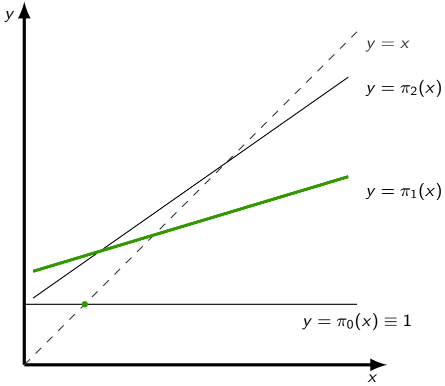

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- Not inductive: $f(1) > 1$

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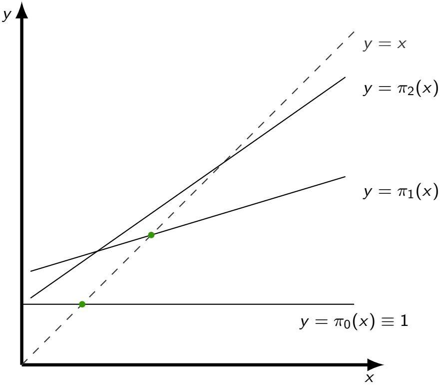

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- Value evaluates to 1.8: $\max d \text{ s.t. } d \leq \pi_2(d)$

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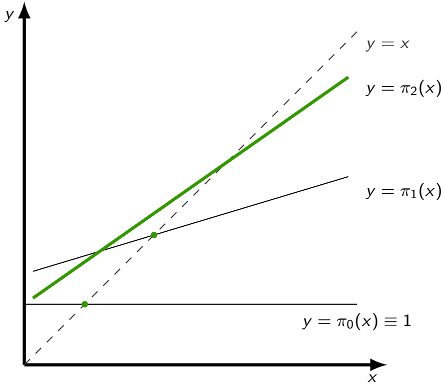

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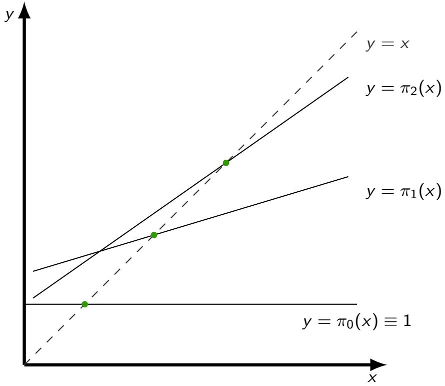

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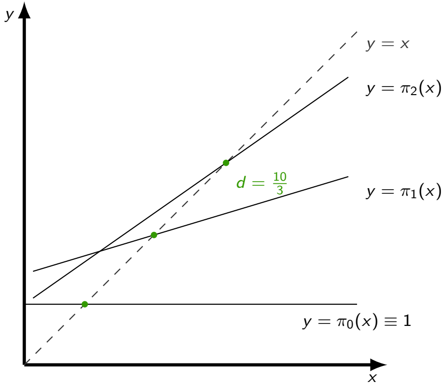

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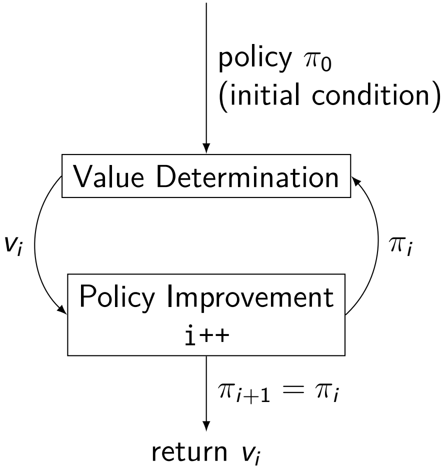
- Not inductive: $f(1.8) > 1.8$
- Value evaluates to $\frac{10}{3}$: $\max d$ s.t. $d \leq \pi_3(d)$
- Inductive!: $f(\frac{10}{3}) = \frac{10}{3}$

Policy Iteration

Iteration Algorithm



- Iterate on policies
- Find value for each
- Terminate on inductiveness

Max-Policy Iteration

Larger Programs

- Multiple templates, multiple nodes:
  - Choice of incoming transition per template, per node

Max-Policy Iteration

Larger Programs

- Multiple templates, multiple nodes:
  - Choice of incoming transition per template, per node
- Policy Improvement: SMT query
- Value Determination: LP query

Outline

Problems of Policy Iteration

Motivation for our work

Policy Iteration is under-determined

Which policy to improve? When? How?

- Arising issues:
  - Scalability: solving global equation system
  - Iteration Order: not defined in the algorithm
  - Cooperability: doesn't fit into existing frameworks

Local Policy Iteration

Integrating Abstract Interpretation

- Our work: LPI (Local Policy Iteration)
  - Exploits existing iteration strategies
  - Avoids solving the global equation at each step
  - Unifies policy iteration: precise widening operator

Idea

Bring results from abstract interpretation back into policy iteration (iteration order, locality, communication)

Abstract Interpretation Formulation

Required Ingredients

Abstract Interpretation Formulation

Required Ingredients

- Abstract domain: $\mathcal{D}$, partial order $\sqsubseteq$
- Join operator: $\sqcup : \mathcal{D} \rightarrow \mathcal{D} \rightarrow \mathcal{D}$
- Postcondition operator: $\rightsquigarrow: \mathcal{D} \rightarrow \tau(\mathbf{x} \cup \mathbf{x}') \rightarrow \mathcal{D}$
- Widening $\nabla : \mathcal{D} \rightarrow \mathcal{D} \rightarrow \mathcal{D}$

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Aim

Express policy iteration as classical Kleene iteration

LPI Abstract Domain $\mathcal{D}$

Idea

Set of reachable abstract states stores policy implicitly

LPI Abstract Domain $\mathcal{D}$

Idea

Set of reachable abstract states stores policy implicitly

Definition (LPI State)

- Template constraints domain state + policy information.
- Map from templates to tuples
- (bound $d \in \mathbb{R}$, policy $\pi : \mathbb{R}^n \to \mathbb{R}$, **backpointer** $a \in \mathcal{D}$)

LPI Abstract Domain $\mathcal{D}$

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- Abstract state example: $\{i : (0, i' = 0, \mathbf{A})\}$
- Partial order: component-wise comparison on bounds

LPI Abstract Domain $\mathcal{D}$

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- (bound $d \in \mathbb{R}$, policy $\pi : \mathbb{R}^n \to \mathbb{R}$, **backpointer** $a \in \mathcal{D}$)

- Abstract state example: $\{i : (0, i' = 0, \mathbf{A})\}$
- Partial order: component-wise comparison on bounds

- For mapping $s \equiv \{t : (d, \pi, a)\}$
  - Concretization γ: $\{\mathbf{x} \mid t^\top \mathbf{x} \leq d\}$
  - Invariant: $d = \pi(\gamma(a))$

LPI Postcondition Computation

$$a_0 \equiv \{x : (4, \ldots, \ldots)\} \xrightarrow{x' = x + 1 \lor x' = 2x} \; ?$$

- Record the policy and the backpointer along with the bound
- Policy: concave under-approximation of transition relation
- Computed using optimization modulo SMT:
  - Successor is $a_1 \equiv \{x : (8, \lambda d.\max x' \text{ s.t. } x' = 2x \land x \le d, a_0)\}$

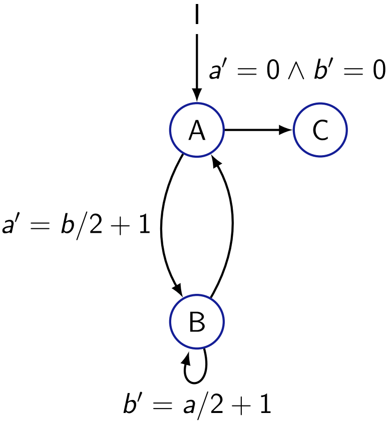
Policy Improvement

Implicitly chooses best policy locally

Example

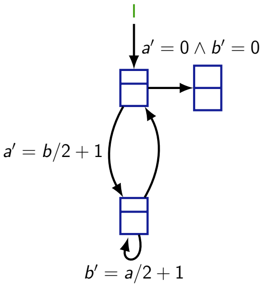
Applying LPI algorithm

```
double a = 0, b = 0;
while (input()) {
    a = b / 2 + 1;
    while (input()) {
        b = a / 2 + 1;
    }
}
```



Local Policy Iteration

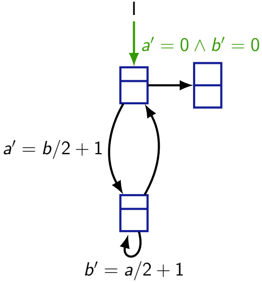
Algorithm Example



1. Start with $\top$ state a_0

Local Policy Iteration

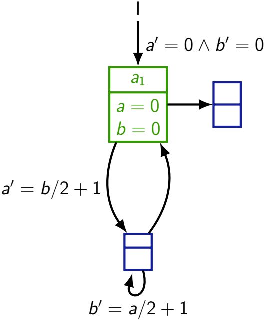
Algorithm Example



1. Start with $\top$ state a_0
2. Postcondition generates new state a_1

Local Policy Iteration

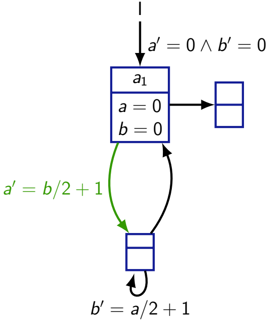
Algorithm Example



1. Start with $\top$ state a_0
2. Postcondition generates new state a_1
3. Associate a_1 with node A

Local Policy Iteration

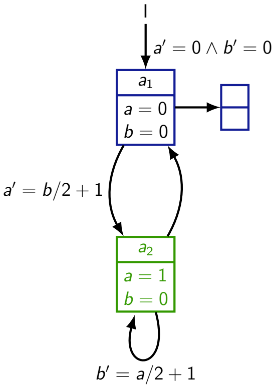
Algorithm Example



1. Start with $\top$ state a_0
2. Postcondition generates new state a_1
3. Associate a_1 with node A
4. Postcondition generates new state a_2

Local Policy Iteration

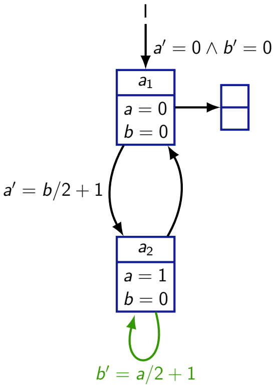
Algorithm Example



1. Start with $\top$ state a_0
2. Postcondition generates new state a_1
3. Associate a_1 with node A
4. Postcondition generates new state a_2
5. Associate a_2 with node B

Local Policy Iteration

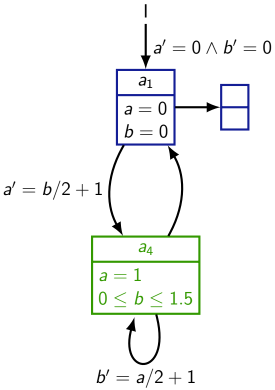
Algorithm Example



1. Start with $\top$ state a_0
2. Postcondition generates new state a_1
3. Associate a_1 with node A
4. Postcondition generates new state a_2
5. Associate a_2 with node B
6. Postcondition generates a_3

Local Policy Iteration

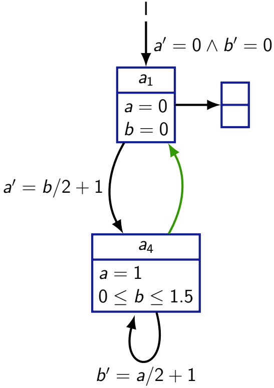
Algorithm Example



1. Start with $\top$ state a_0
2. Postcondition generates new state a_1
3. Associate a_1 with node A
4. Postcondition generates new state a_2
5. Associate a_2 with node B
6. Postcondition generates a_3
7. Join a_2 and a_3, run subsequent value determination, associate result a_4 with B

Local Policy Iteration

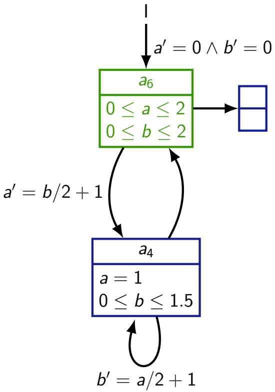
Algorithm Example



1. Start with $\top$ state a_0
2. Postcondition generates new state a_1
3. Associate a_1 with node A
4. Postcondition generates new state a_2
5. Associate a_2 with node B
6. Postcondition generates a_3
7. Join a_2 and a_3, run subsequent value determination, associate result a_4 with B
8. Postcondition generates a_5

Local Policy Iteration

Algorithm Example



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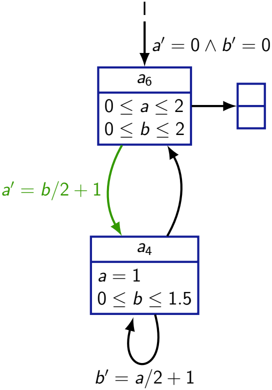
3. Associate a_1 with node A
4. Postcondition generates new state a_2
5. Associate a_2 with node B
6. Postcondition generates a_3
7. Join a_2 and a_3, run subsequent value determination, associate result a_4 with B
8. Postcondition generates a_5
9. Join a_5 and a_1, subsequent value determination generates a_6, associated with A

Algorithm Example



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8. Postcondition generates a_5
9. Join a_5 and a_1, subsequent value determination generates a_6, associated with A
10. ...

Local Policy Iteration

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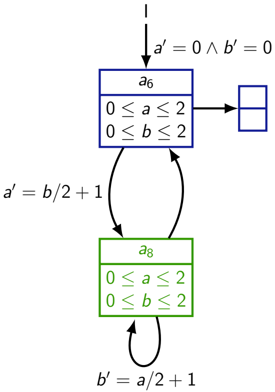

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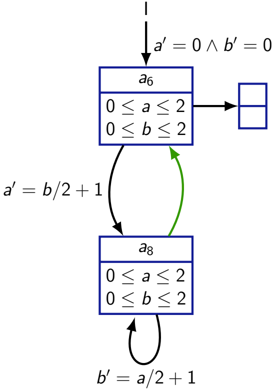

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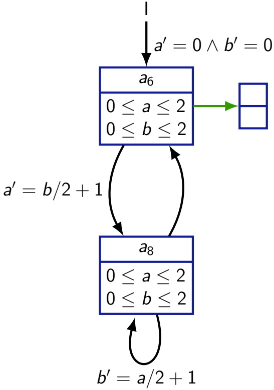

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10. ...

LPI Join Operator

- Joining states a_0 (previous), and a_1 (new)
- For each template $t \in T$:
  - Keep $a_0[t]$, unless bound in $a_1[t]$ is strictly larger
  - Guarantees feasibility
- If variable dependencies form a strongly connected component:
  - Launch value determination

Result

Together with postcondition simulates policy improvement

LPI Value Determination

- On updated templates:

LPI Value Determination

- On updated templates:
  - Reconstruct the equation system using the recorded policies
  - Recover the strongly connected component of variable dependencies
  - Solve LP to find the policy value

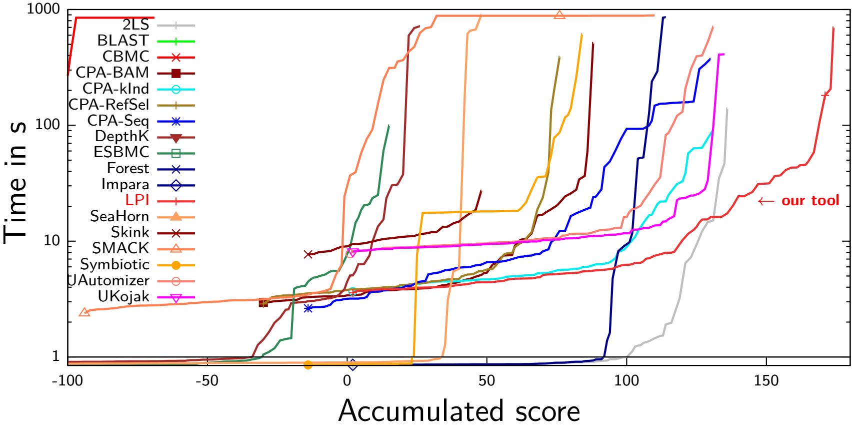
Value Determination Problem

Potentially size of the largest loop

SV-COMP Results 2016

Evaluation

First in Loops category!



Conclusion

LPI Features

- Local updates
- Update frontier
- Iteration order
  - Can use existing results
- Fits into existing frameworks
- Can be run in parallel with other analyses

LPI Formulation

Precise widening operator converging in finite number of steps

Outline

Template Generation Strategies

Annotations defining domain *shape*

- Combinatorial Synthesis
- Abstract reachability tree generation
  - Enables counterexample traces
  - Enables interpolation
- Synthesis using polyhedral analysis

  Generating templates using convex hull and projection
  - Offline
    - Generate templates using convex hull, use after restart
  - Online
    - Value determination before widening in polyhedral abstract interpretation

Underlying Theme

Refine template size on failed analysis

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Underlying Theme

Refine template size on failed analysis

Combinatorial Enumeration

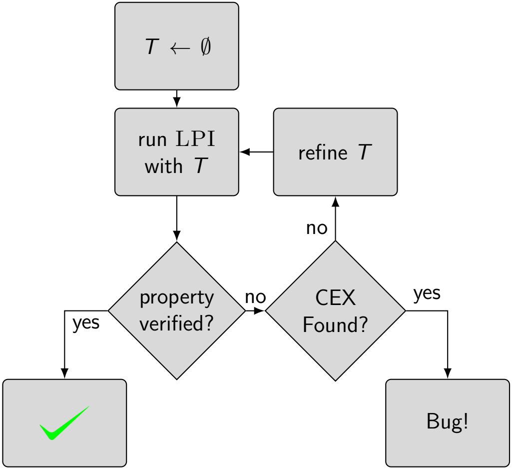
- Defining set of templates
  - Fix occurring constants (say, 1)
  - Fix expression size
  - E.g. vars: `int x, y`, size: 1, constants: $\{1, 0\}$
    - Generates $\{x, y\}$
- Refinement
  - raise the expression size
  - allow more constants
- Upper size bound: # of variables

Example

- For two variables x, y:

$\emptyset, \{x, y\}, \{x + y, x - y, -x - y, y - x\}, \ldots$

Combinatorial Enumeration



- Liveness & redundancy filtering
- Good results in practice

Abstract Reachability Tree Generation

- Goal: construct using abstract interpretation

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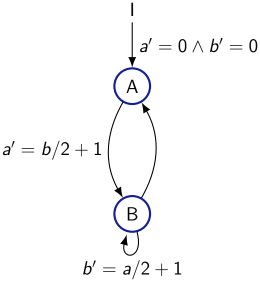
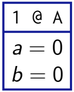
Tree Construction

- Enables counterexample traces
- Enables interpolation

ART Generation Example





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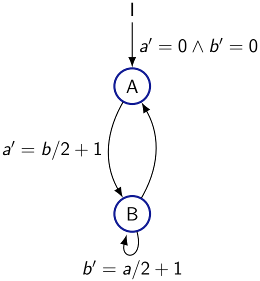

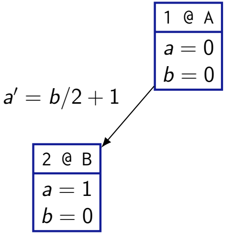

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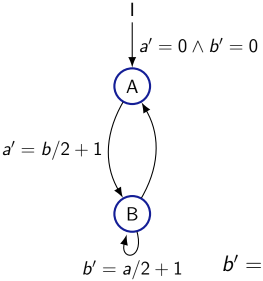

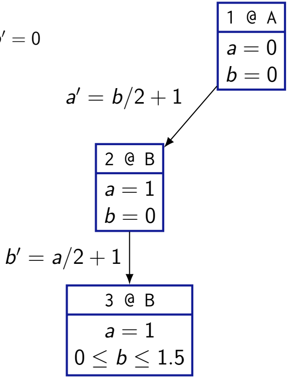

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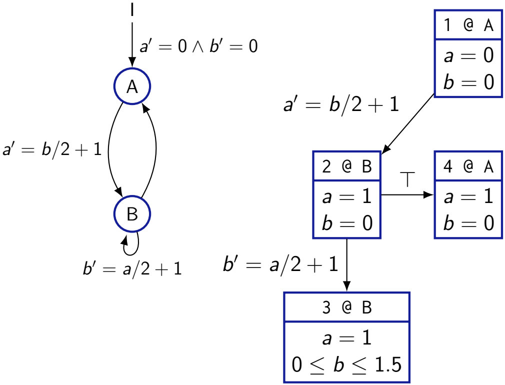

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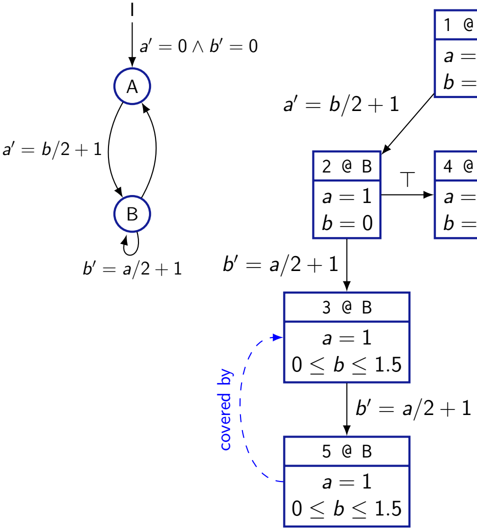

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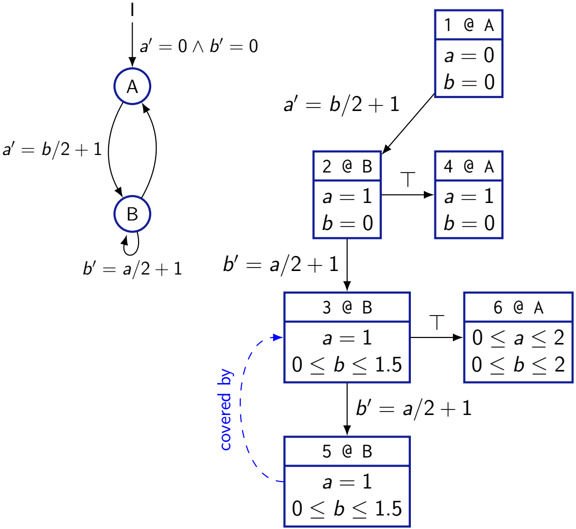

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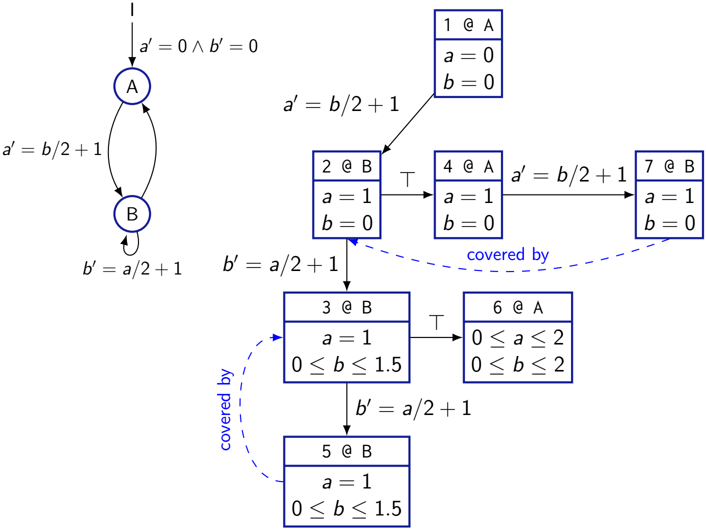

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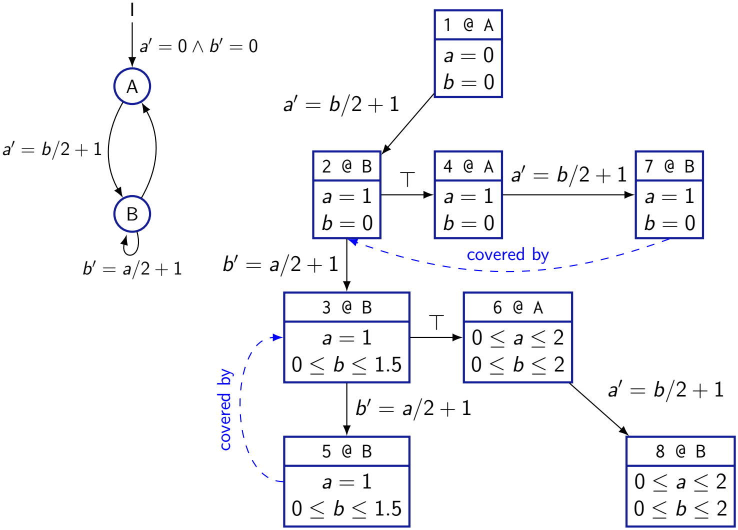

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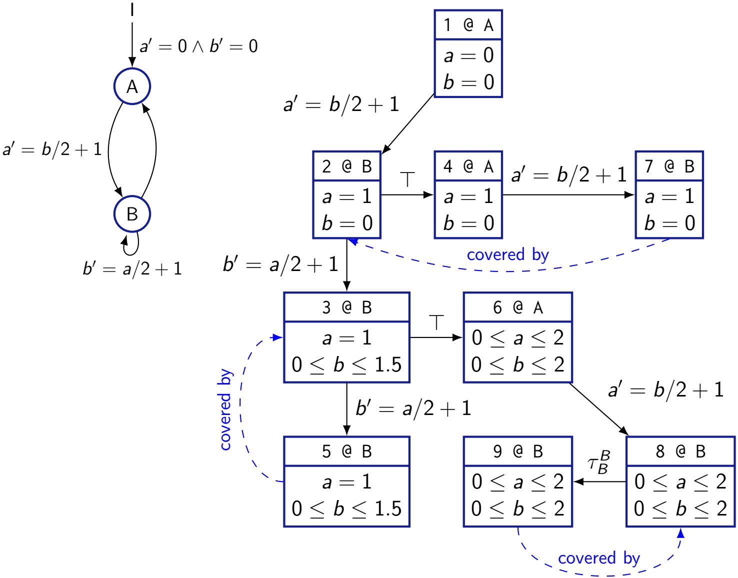

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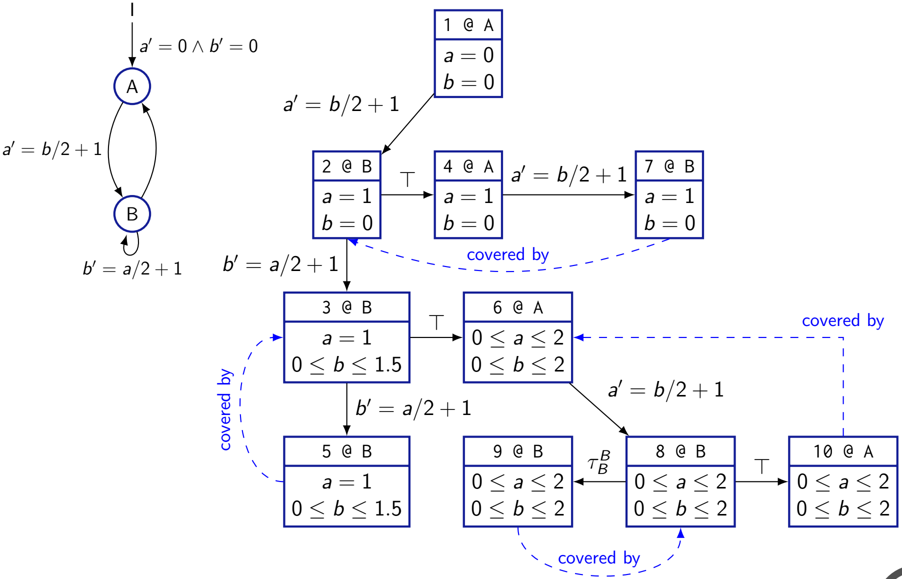

Outline

Summary Generation

- One-state invariants $P(\mathbf{x})$:
  - Inlining is exponential
  - No recursion support
- Solution: summarize functions
  - Invariants $S(\mathbf{x} \cup \mathbf{x}')$
  - Possible transitions within the function...
  - ...with valid calling context

Summary Equations

$$
\begin{aligned}
\text{Program Initiation: } & I_{f_m}^{m} = \top \\
\text{Consecution: } & \text{for all } (a, \texttt{OPS}, b) \in \mathit{edges}: \\
& [\![\texttt{OPS}]\!]^{\sharp}(I_f^a) \preceq I_f^b \\
\text{Function Call: } & \text{for all } (g, n_{call}, n_{ret}, \mathbf{x}_p, \mathbf{x}_o) \in \mathit{calledges}: \\
& I_f^{n_{call}}|_{\mathbf{x}_p}[\mathbf{x}_p / \mathbf{x}_i^g] \preceq I_f^{n_{en}} \\
\text{Summary Coverage: } & I_f^{n_{ex}}|_{\mathbf{x}_i \cup \mathbf{x}_r} \preceq S_f \\
\text{Function Application: } & \text{for all } (g, n_{call}, n_{ret}, \mathbf{x}_p, \mathbf{x}_o) \in \mathit{calledges}: \\
& I_f^{n_{call}}|_{\mathbf{x} \setminus \mathbf{x}_o} \sqcap S_g[\mathbf{x}_i^g / \mathbf{x}_p][\mathbf{x}_r^g / \mathbf{x}_o] \preceq I_f^{n_{ret}}
\end{aligned}
$$

Contribution

Generating least inductive summaries using policy iteration

Inductive Invariants from Preconditions

Formula Slicing

- Verification: loop-free program fragments can be exactly encoded as formulas

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```
struct vmxnet3_adapter *adapter = netdev_priv(netdev);
u32 *buf = p;
int i = 0, j = 0;
memset(p, 0, vmxnet3_get_regs_len(netdev));
regs->version = 2;
buf[j++] = VMXNET3_READ_BAR1_REG(adapter, REG_VRRS);
buf[j++] = VMXNET3_READ_BAR1_REG(adapter, REG_UVRS);
buf[j++] = VMXNET3_READ_BAR1_REG(adapter, REG_DSAL);
// ...
buf[j++] = adapter->intr.num_intrs;
for (i = 0; i < adapter->intr.num_intrs; i++) {
    buf[j++] = VMXNET3_READ_BAR0_REG(adapter, ...);
}
```

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Our Solution

For program trace find inductive over-approximation over the loop

JavaSMT Library

- Satisfiability modulo theories solvers:
  - Ubiquitous in program analysis
- SMT-LIB initiative: often limited
- Solver API: vendor lock-in
- Solution:
  - Common API for using SMT solvers
  - Proper types, introspection, performance, etc.

Getting the Library

`https://github.com/sosy-lab/javasmt`

Outline

Conclusion

Contributions Overview

- New algorithms for inductive invariant synthesis
- LPI: unifying policy iteration and abstract interpretation
  - More accessible to engineers
- For all contributions:
  - Implementation in CPAchecker
  - Evaluation

Future Work

Charting the landscape

- Evaluation outside of SV-COMP: towards system verification

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- Integration with model-checking approaches
  - Invariants for predicate analysis
  - Guiding model checking tools

Questions?

Thank you for your time!